Electromagnetic Field Theory

Books:

<u>Electrical</u>

1. Sadiku, Matthew N, "Elements of Electromagnetics", Oxford University Press, ISBN: 0195103688, Latest Edition.
2. William Hayt and John A. Buck, "Engineering Electromagnetics", McGraw-Hill,ISBN: 0073104639, Latest Edition.
3. Kong J. A., "Electromagnetic Wave Theory", Cambridge, Latest Edition.
4. John D. Kraus, "Engineering Electromagnetics", McGraw-Hill Inc., New York, Latest Edition
5. N. N. Rao, "Elements of Engineering Electromagnetics", Pearson Education, Latest Edition

<u>Electronics</u>

1. Electromagnetic waves & radio system by Jorden R.F.
2. Principle and applications of Electromagnetic fields by Ptonsey R and Collin R.P
3. Applied Electromagnetic by Planus M.A.

Fundamentals of Applied Electromagnetics by Fawaz T. Ulaby et. al. latest edition

Distribution of Marks

▪ Assignments	= 12.5 marks
▪ Quizzes	= 12.5 marks
▪ Mid term exam	= 25 marks
▪ Final term exam	= 50 marks

Introduction

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What is Electromagnetic Field Theory ?

- Electromagnetics or Electromagnetism is about the combination of electricity and magnetism
- We know the relation between electricity and magnetism
- Field is any area. In this case, field is the area where the combined effect of electricity and magnetism can be felt
- Theory is simply defined as a system of ideas intended to explain something

Theories of Electromagnetic Field in Real Life ?

- Wireless communication (Antennas)
- RADAR
- Machines and Drives
- Biomedical applications
- Sensors

Short Review of Some Fundamentals

Fundamental SI Units

Dimension	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol

- Derived Units

Prefixes to Represent Numbers/Units

Prefix	Symbol	Magnitude
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

Scalar and Vector Quantities

- Scalar Quantity
  - Physical quantities requiring magnitude for complete description
  - Represented by medium-weighted italic font
- Vector Quantity
  - Physical quantities requiring magnitude and direction for complete description
  - Represented by bold face font (or a normal letter with arrow above it)
  - Magnitude is represented by a medium-weighted italic font
  - Direction is represented by a unit vector (bold face letter with circumflex above it)

$$\mathbf{E} = E\hat{\boldsymbol{x}}$$

The Nature of Electromagnetism

Nature of Electromagnetism

- Electromagnetic force is one of the natures fundamental forces
- It operates at the atomic scale
- Its effects can be transmitted through electromagnetic waves through free space and material media
- Source of electric and magnetic fields
- Combination of electric and magnetic fields (electromagnetic field)

Electric Field

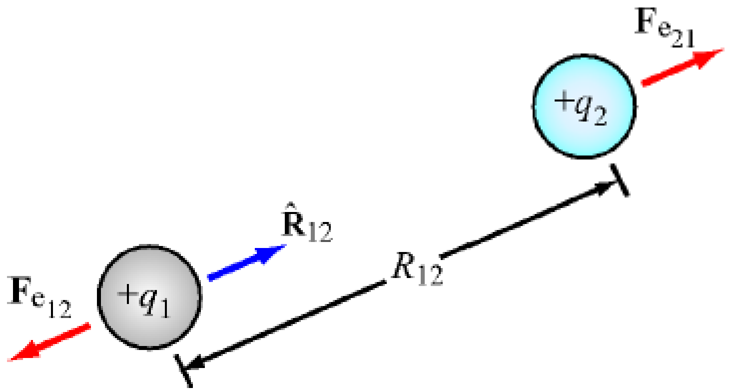
Electric Field

- Source of electric field is electric charge
- Electric charge may have positive or negative polarity
- The resulting force may be attractive or repulsive
- All matter is composed of neutrons (neutral), protons (positively charged), and electrons (negatively charged)
- The fundamental quantity of charge is that of a single electron denoted by e

$$e = 1.6x10^{-19} Coulomb$$

- Charge of a single electron is $q_e = -e$, and that of a proton is $q_p = e$

Coulomb's Law



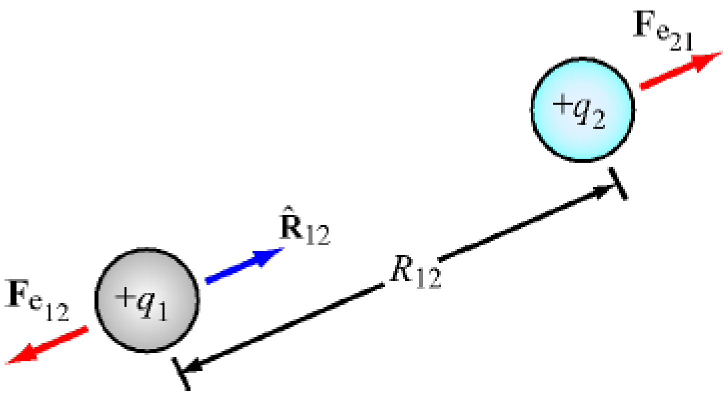
Coulomb's experiments demonstrated that:

(1) *two like charges repel one another, whereas two charges of opposite polarity attract,*

(2) *the force acts along the line joining the charges, and*

(3) *its strength is proportional to the product of the magnitudes of the two charges and inversely proportional to the square of the distance between them.*

Coulomb's Law



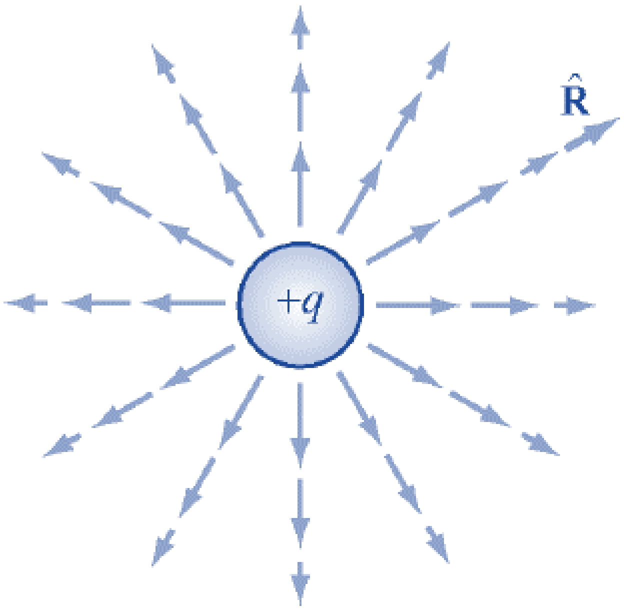
$$\mathbf{F}_{e_{21}} = \hat{\mathbf{R}}_{12} \frac{q_1 q_2}{4\pi \varepsilon_0 R_{12}^2} \quad \text{(N)} \quad \text{(in free space)}$$

- Where F_{e21} is the electric force acting on charge $q2$ due to charge $q1$
- R_{12} is the distance between the two charges
- $\widehat{R_{12}}$ is the unit vector pointing from $q1$ to $q2$
- ε_0 is a universal constant called the electrical permittivity of free space

$$\varepsilon_0 = 8.854 \text{ x } 10^{-12} F/m$$

- $F_{e12} = -F_{e21}$

Electric Field Intensity



$$\mathbf{E} = \hat{\mathbf{R}} \frac{q}{4\pi \varepsilon_0 R^2} \quad \text{(V/m)} \quad \text{(in free space)},$$

If any point charge q' is present in an electric field $\mathbf{E}$ *(due to other charges), the point charge will experience a force acting on it equal to* $\mathbf{F}_e = q'\mathbf{E}$.

Electric Field Intensity

- The electric field intensity, due to any charge q, can be defined as

$$\mathbf{E} = \hat{\mathbf{R}} \frac{q}{4\pi\varepsilon_0 R^2} \quad \text{(V/m)} \quad \text{(in free space)},$$

If any point charge q' is present in an electric field $\mathbf{E}$ (due to other charges), the point charge will experience a force acting on it equal to $\mathbf{F}_e = q'\mathbf{E}$.

- Where, R is the distance between charge and the point of observation
- $\hat{R}$ is the unit vector pointing away from the charge

Two Important Properties of Electric Charge

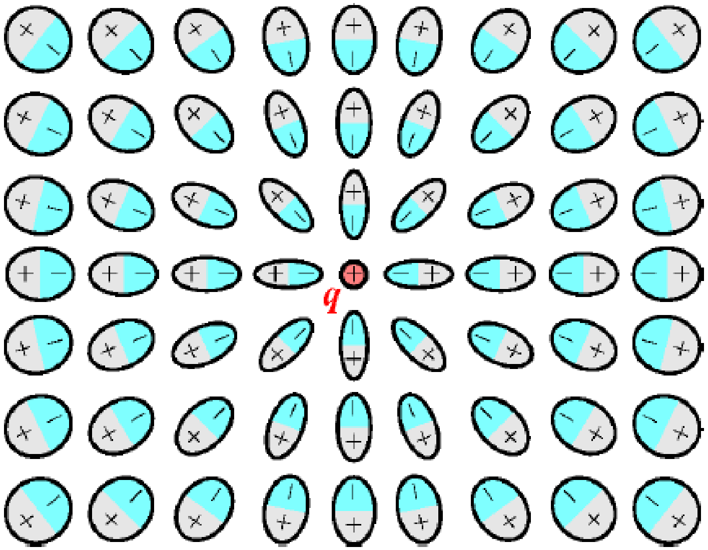
1. Law of Conservation of Charge: The electric charge (net) can neither be created nor destroyed

   | if a volume contains n_e number of electrons and n_p number of protons, then the total charge q is given by $q = n_p e - n_e e = (n_p - n_e)e$

2. Principle of linear superposition: the total vector electric field at a point in space due to a system of point charges is equal to the vector sum of electric fields at that point due to the individual fields.

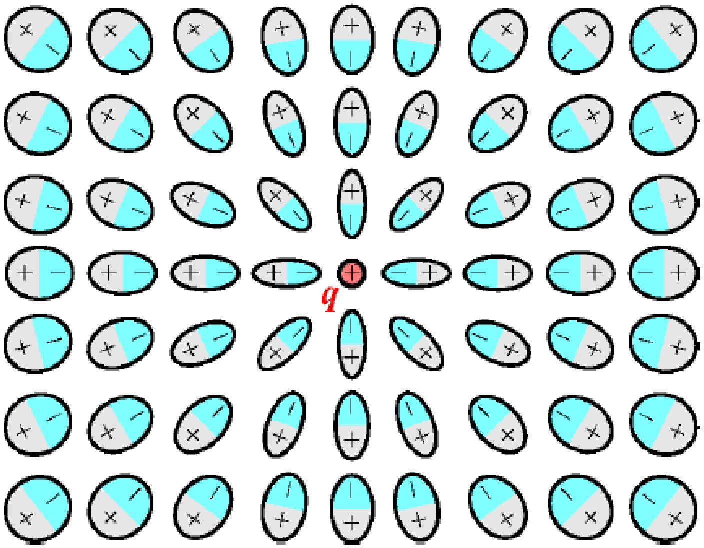
   | Polarization of atoms of a dielectric material:

- Polarization of atoms of a dielectric material due to a positive charge q: Under normal conditions, electric field at any point inside a molecule is zero. When a positive charge q is placed inside the material, the electrons of all the atoms get attracted to q, hence giving rise to electric dipoles. The overall process of electric dipole creation and their alignment is called polarization.



Electric Dipole and Polarization

- Electrons exist around the nucleus of atoms under normal conditions, but if due to any external force the electrons get concentrated on one side, leaving the rest of the charge on other side, such a polarized atom is known as electric dipole
- The intensity of polarization depends upon the distance of the atom from the test charge



Permittivity and Relative Permittivity of Material

- The electric field intensity at a point near a test charge, is different in vacuum as compared to that in a material. It is given by

$$\mathbf{E} = \hat{\mathbf{R}} \frac{q}{4\pi\varepsilon R^2} \qquad \text{(V/m)}$$

- *ε is the permittivity of the material*
- Normally the permittivity of different materials is given with reference to that of free space as $\varepsilon = \varepsilon_r \varepsilon_0 \ (F/m)$, where ε_r is the relative permittivity of that material
- For vacuum ε_r=1, while for air near earth's surface ε_r=1.0006

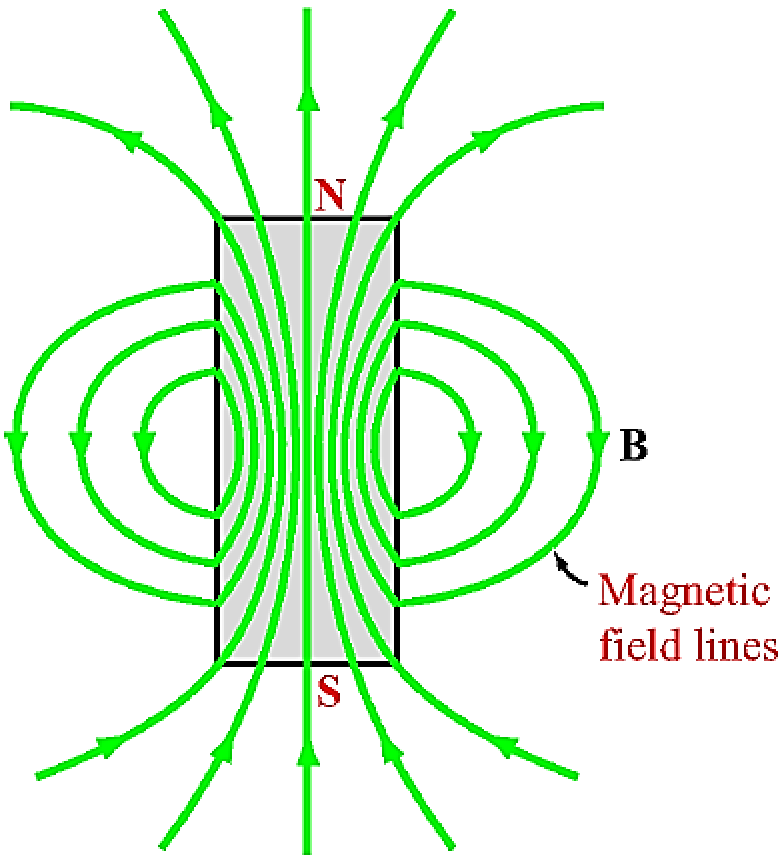
Electric Flux Density

$$D = \varepsilon E \quad (C/m^2)$$

Magnetic Field

Magnet and Magnetic Field

- Magnets occurred naturally
- The idea of magnetic field lines, their direction, and poles was presented using the compass needle as a testing instrument



Magnet and Magnetic Field

- Magnetic field lines surrounding the magnet shows the magnetic flux density represented by B
- Magnetic field is also generated by the flow of electric charge.
- The following relation, known as Biot-savart law, describes the magnitude of magnetic flux density around a current carrying conductor

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r} \qquad \text{(T)}$$

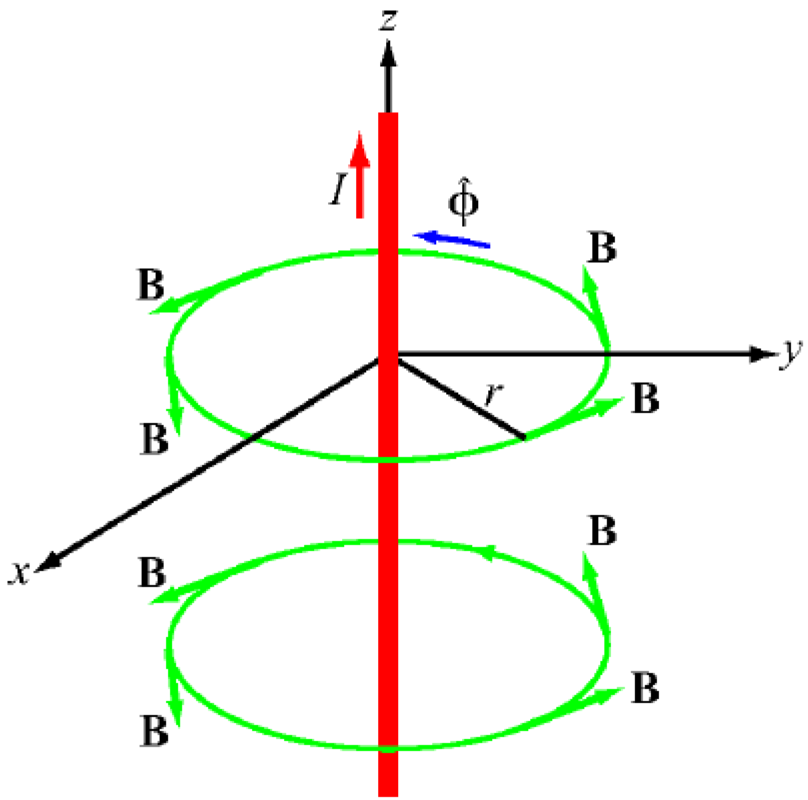
- μ_0 is called the magnetic permeability of free space and its value is

$$4\pi \text{ x } 10^{-7} \ H/m$$

It is analogous to the electric permittivity ε_0

Biot-Savart Law

- The magnetic field induced by a steady state current flowing in the z-direction



$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r} \qquad \text{(T)}$$

Magnetic Permeability

- As discussed for electric permittivity, for magnetic permeability we have

$$\mu = \mu_r \mu_0$$

- Also, in analogy to the relation between electric field intensity and electric flux density, we have

$$B = \mu H$$

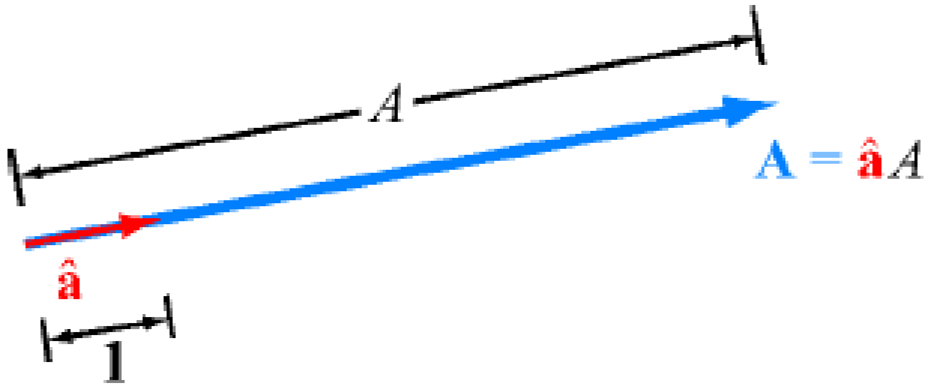
- Where B is the magnetic flux density, while H is the magnetic field intensity

Vector Analysis

Vector Analysis

- Vector representation
- Need of vector analysis in EMF

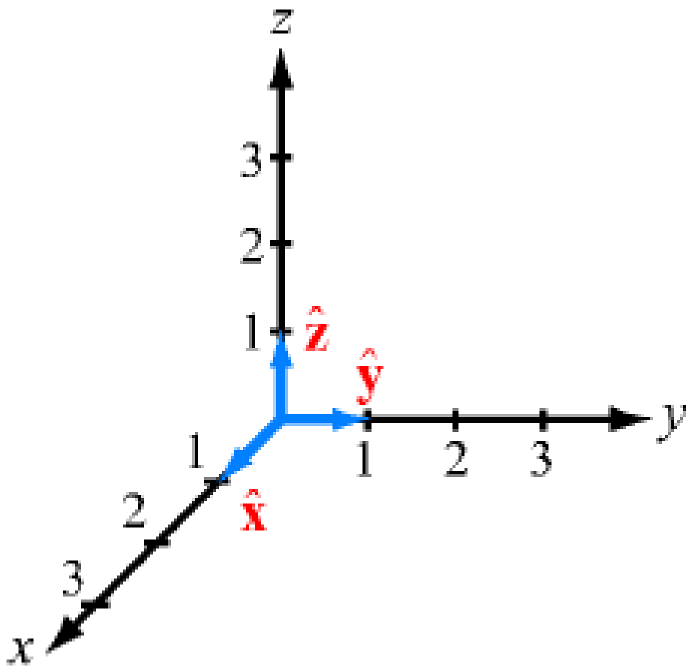
- Appearance of vector
- Magnitude and direction of vector
- Unit vector



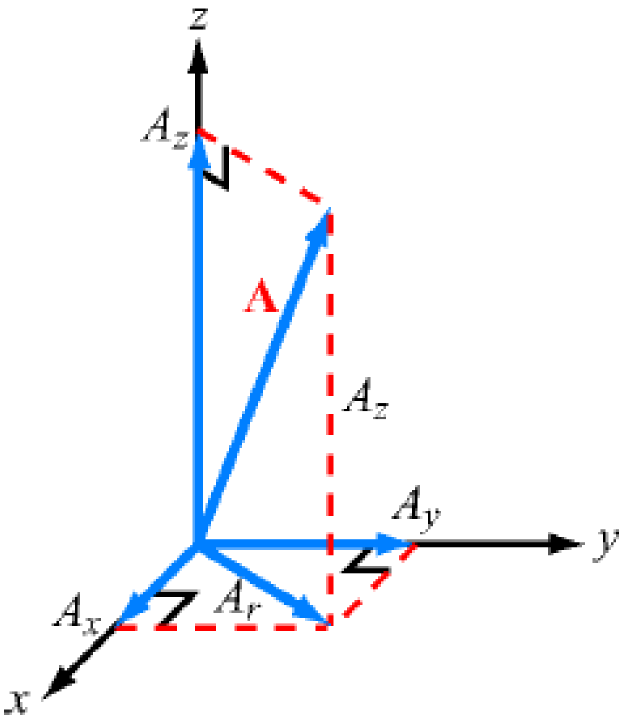
$$\mathbf{A} = \hat{\mathbf{a}}|\mathbf{A}| = \hat{\mathbf{a}}A$$

$$\hat{\mathbf{a}} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}$$

Base Vectors and Components of Vector in Cartesian Coordinate System



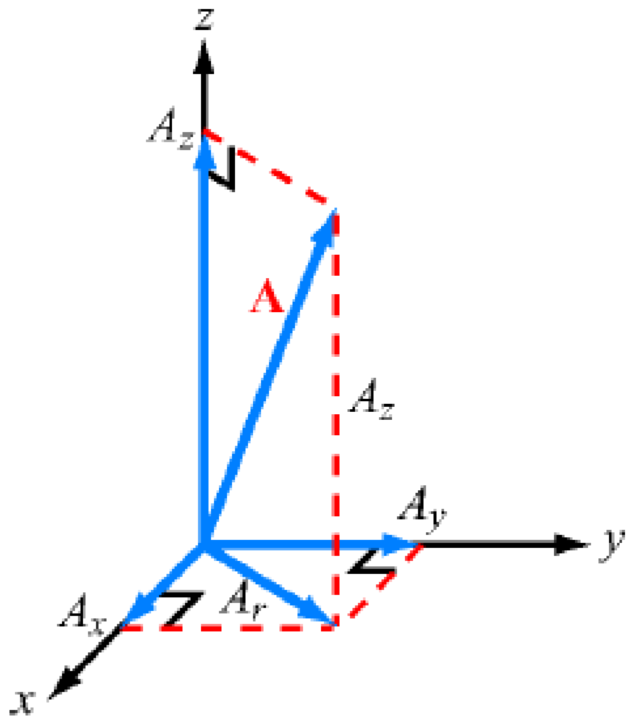
Base Vectors



Components of a Vector

$$\mathbf{A} = \hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$$

Components of Vector in Cartesian Coordinate System



Components of a Vector

$$\mathbf{A} = \hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$$

$$A = |\mathbf{A}| = \sqrt[+]{A_x^2 + A_y^2 + A_z^2}$$

$$\hat{\mathbf{a}} = \frac{\mathbf{A}}{A} = \frac{\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z}{\sqrt[+]{A_x^2 + A_y^2 + A_z^2}}$$

Equality of Two Vectors

$$\mathbf{A} = \hat{\mathbf{a}}A = \hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z,$$

$$\mathbf{B} = \hat{\mathbf{b}}B = \hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z,$$

then $\mathbf{A} = \mathbf{B}$ if and only if $A = B$ and $\hat{\mathbf{a}} = \hat{\mathbf{b}}$, which requires that $A_x = B_x$, $A_y = B_y$, and $A_z = B_z$.

Equality of two vectors does not necessarily imply that they are identical; in Cartesian coordinates, two displaced parallel vectors of equal magnitude and pointing in the same direction are equal, but they are identical only if they lie on top of one another.

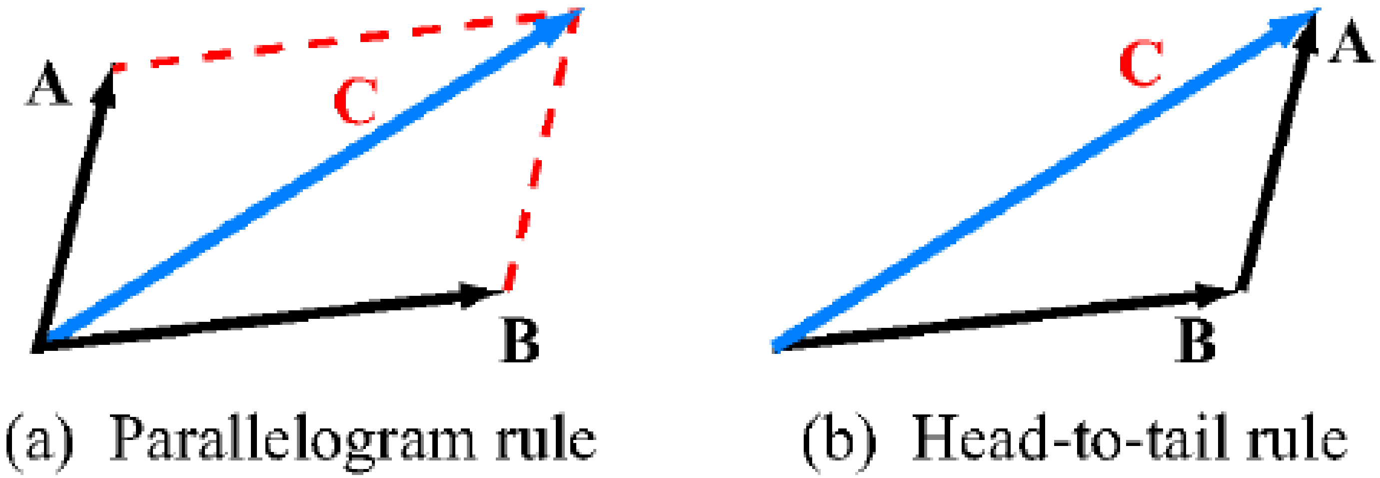
The sum of two vectors $\mathbf{A}$ and $\mathbf{B}$ is a vector $\mathbf{C} = \hat{\mathbf{x}}\,C_x + \hat{\mathbf{y}}\,C_y + \hat{\mathbf{z}}\,C_z$, given by

$$\begin{aligned}
\mathbf{C} &= \mathbf{A} + \mathbf{B} \\
&= (\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z) + (\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z) \\
&= \hat{\mathbf{x}}(A_x + B_x) + \hat{\mathbf{y}}(A_y + B_y) + \hat{\mathbf{z}}(A_z + B_z) \\
&= \hat{\mathbf{x}}\,C_x + \hat{\mathbf{y}}\,C_y + \hat{\mathbf{z}}\,C_z.
\end{aligned}$$

Hence, vector addition is commutative:

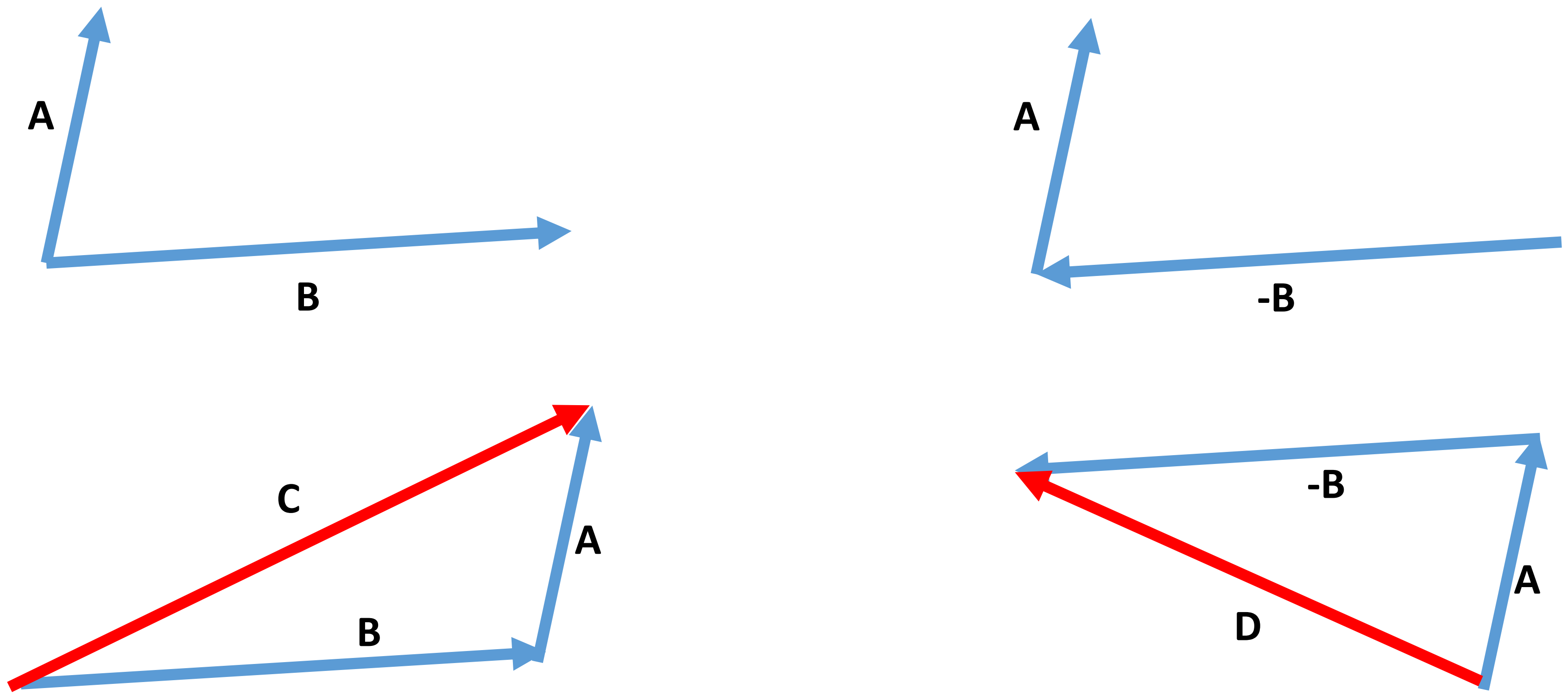
$$\mathbf{C} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}.$$

Addition of Two Vectors (Graphically)



Subtraction of Vectors

$$
\begin{aligned}
\mathbf{D} &= \mathbf{A} - \mathbf{B} \\
&= \mathbf{A} + (-\mathbf{B}) \\
&= \hat{\mathbf{x}}(A_x - B_x) + \hat{\mathbf{y}}(A_y - B_y) + \hat{\mathbf{z}}(A_z - B_z)
\end{aligned}
$$



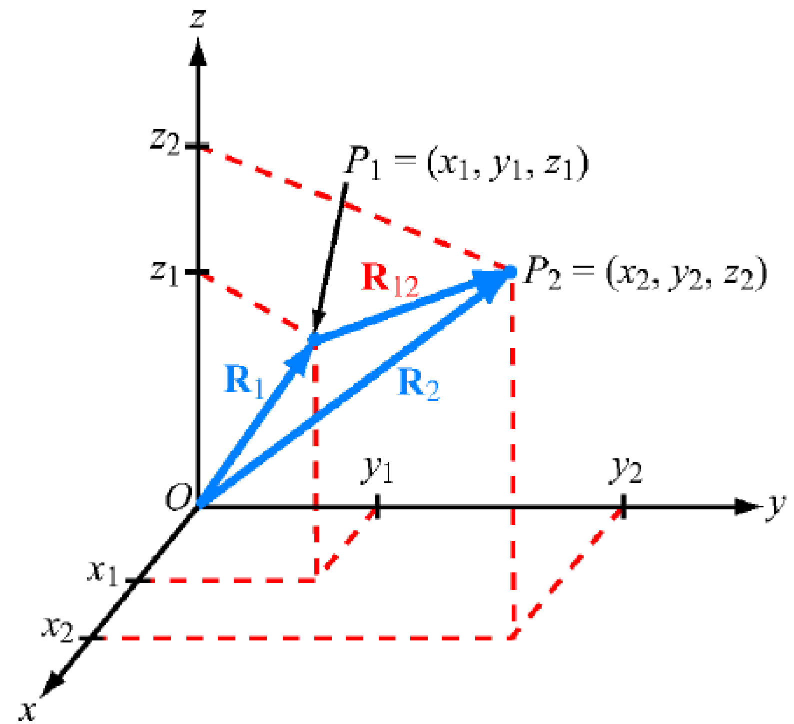
Position and Distance Vectors

- Position vector of a point P in space is a vector from origin to point P

$$\mathbf{R}_1 = \overrightarrow{OP_1} = \hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1,$$
$$\mathbf{R}_2 = \overrightarrow{OP_2} = \hat{\mathbf{x}}x_2 + \hat{\mathbf{y}}y_2 + \hat{\mathbf{z}}z_2,$$

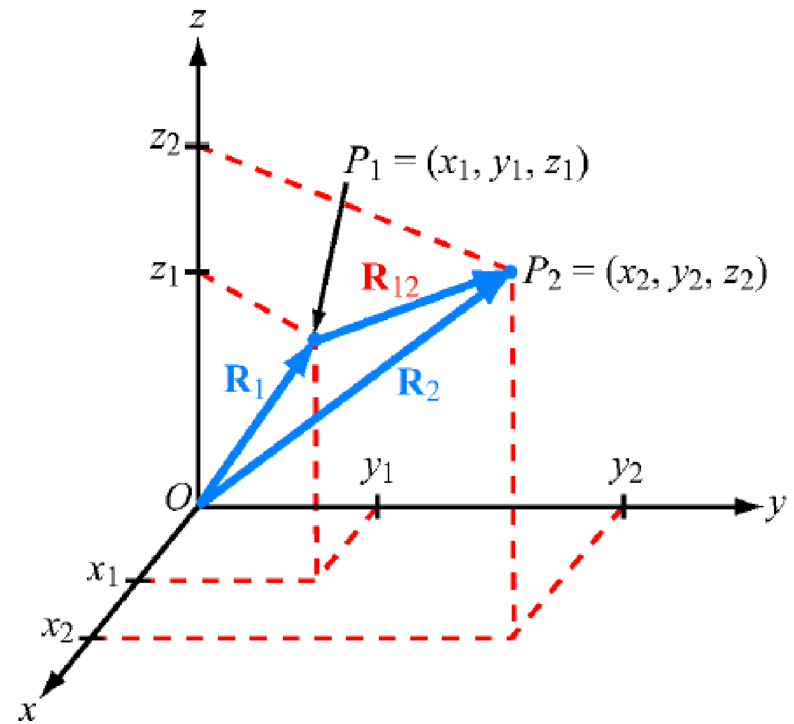
- Distance vector from $P1$ to $P2$ is defined as

$$\begin{aligned}\mathbf{R}_{12} &= \overrightarrow{P_1P_2} \\ &= \mathbf{R}_2 - \mathbf{R}_1 \\ &= \hat{\mathbf{x}}(x_2 - x_1) + \hat{\mathbf{y}}(y_2 - y_1) + \hat{\mathbf{z}}(z_2 - z_1),\end{aligned}$$



- The distance d between $P1$ and $P2$ equals the magnitude of $R12$

$$d = |\mathbf{R}_{12}|$$
$$= [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^{1/2}.$$



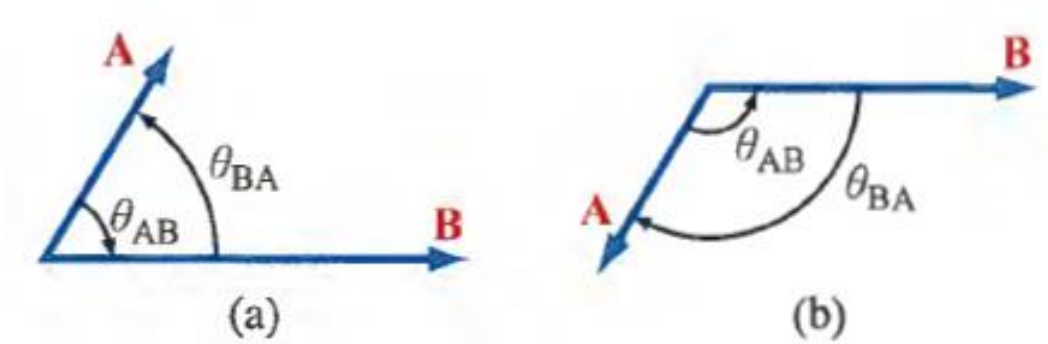
Vector Multiplication

- Simple Product: multiplying a constant k to a vector **A** to get vector **B**

$$\mathbf{B} = k\mathbf{A} = \hat{\mathbf{a}}kA = \hat{\mathbf{x}}(kA_x) + \hat{\mathbf{y}}(kA_y) + \hat{\mathbf{z}}(kA_z)$$
$$= \hat{\mathbf{x}}\,B_x + \hat{\mathbf{y}}\,B_y + \hat{\mathbf{z}}\,B_z.$$

- Scalar Product/dot product:

$$\boxed{\mathbf{A} \cdot \mathbf{B} = AB\cos\theta_{AB}}$$



Properties of Dot Product

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A} \quad \text{(commutative property)},$$

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C} \quad \text{(distributive property)}.$$

$$\mathbf{A} \cdot \mathbf{A} = |\mathbf{A}|^2 = A^2$$

$$A = |\mathbf{A}| = \sqrt[+]{\mathbf{A} \cdot \mathbf{A}}$$

$$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1,$$
$$\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{x}} = 0.$$

If $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$, then

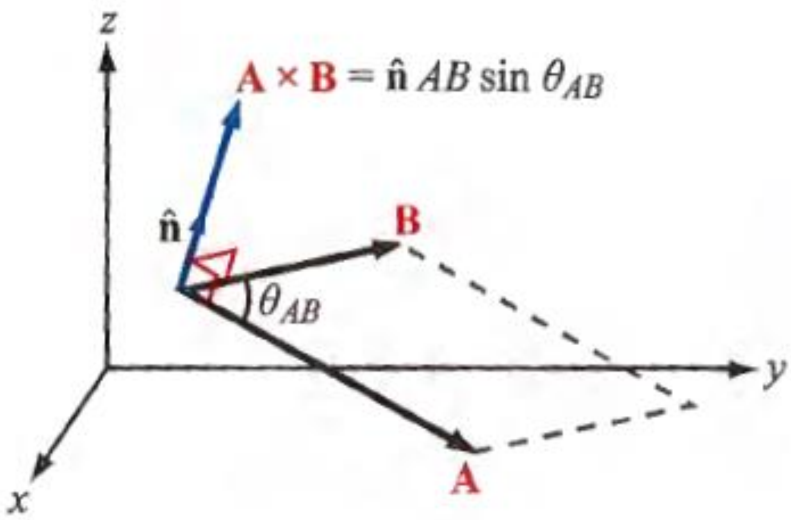
$$\mathbf{A} \cdot \mathbf{B} = (\hat{\mathbf{x}} A_x + \hat{\mathbf{y}} A_y + \hat{\mathbf{z}} A_z) \cdot (\hat{\mathbf{x}} B_x + \hat{\mathbf{y}} B_y + \hat{\mathbf{z}} B_z)$$

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z.$$

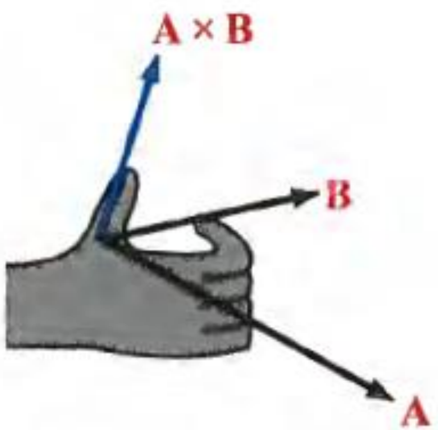
Vector/Cross Product:

$$\mathbf{A} \times \mathbf{B} = \hat{\mathbf{n}}\, AB \sin\theta_{AB}$$

where $\hat{\mathbf{n}}$ is a ***unit vector normal to the plane containing* A *and* B**



Cross product



Right-hand rule

Vector Multiplication

Properties of Vector/Cross Product:

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \qquad \text{(anticommutative)}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C} \quad \text{(distributive)},$$

$$\mathbf{A} \times \mathbf{A} = 0.$$

$$\hat{x} \times \hat{y} = \hat{z} \qquad \hat{y} \times \hat{z} = \hat{x} \qquad \hat{z} \times \hat{x} = \hat{y}$$

$$\boxed{\hat{\mathbf{x}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}} \times \hat{\mathbf{z}} = 0}$$

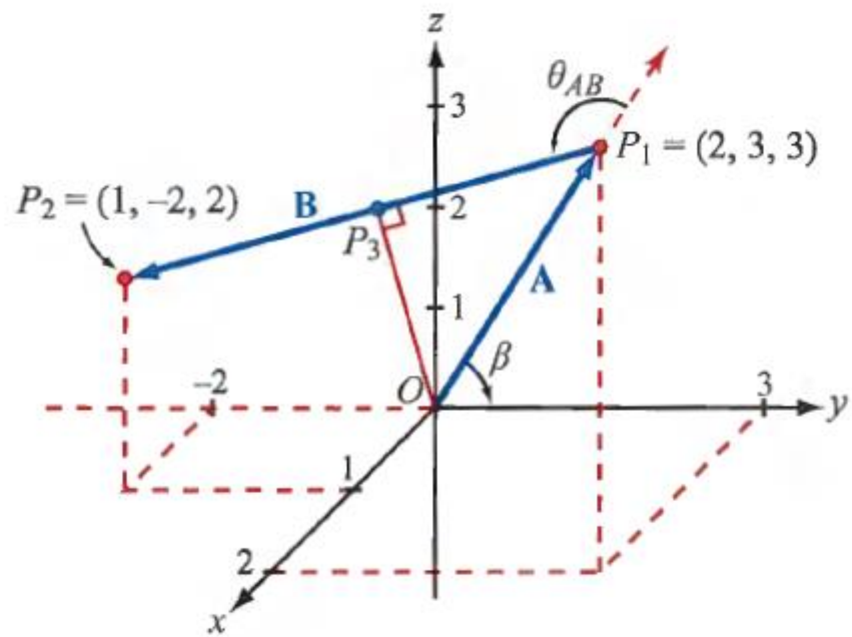
Vector Multiplication

Properties of Vector/Cross Product:

$$\mathbf{A} \times \mathbf{B} = (\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z) \times (\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z)$$
$$= \hat{\mathbf{x}}(A_yB_z - A_zB_y) + \hat{\mathbf{y}}(A_zB_x - A_xB_z)$$
$$+ \hat{\mathbf{z}}(A_xB_y - A_yB_x).$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Example



a) Vector $\mathbf{A}$*, its magnitude, and a unit vector in the direction of* $\mathbf{A}$

$$\mathbf{A} = \hat{\mathbf{x}}2 + \hat{\mathbf{y}}3 + \hat{\mathbf{z}}3,$$

$$A = |\mathbf{A}| = \sqrt{2^2 + 3^2 + 3^2} = \sqrt{22}\,,$$

$$\hat{\mathbf{a}} = \frac{\mathbf{A}}{A} = (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}3 + \hat{\mathbf{z}}3)/\sqrt{22}\,.$$

Example

b) $angle\ \beta\ between\ Vector\ \mathbf{A}\ and\ y - axis$

$$\mathbf{A} \cdot \hat{\mathbf{y}} = |\mathbf{A}||\hat{\mathbf{y}}| \cos\beta = A\cos\beta,$$

or

$$\beta = \cos^{-1}\left(\frac{\mathbf{A} \cdot \hat{\mathbf{y}}}{A}\right) = \cos^{-1}\left(\frac{3}{\sqrt{22}}\right) = 50.2^\circ$$

c) $Vector\ \mathbf{B}$

$$\mathbf{B} = \hat{\mathbf{x}}(1-2) + \hat{\mathbf{y}}(-2-3) + \hat{\mathbf{z}}(2-3) = -\hat{\mathbf{x}} - \hat{\mathbf{y}}5 - \hat{\mathbf{z}}.$$

d) $Angle\ \theta_{AB}\ between\ \mathbf{A}\ and\ \mathbf{B}$

$$\theta_{AB} = \cos^{-1}\left[\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}\right] = \cos^{-1}\left[\frac{(-2-15-3)}{\sqrt{22}\sqrt{27}}\right]$$

$$= 145.1^\circ.$$

Example

e) *the perpendicular distance from origin to vector B*

$$|\overrightarrow{OP_3}| = |\mathbf{A}| \sin(180^\circ - \theta_{AB})$$
$$= \sqrt{22}\, \sin(180^\circ - 145.1^\circ) = 2.68.$$

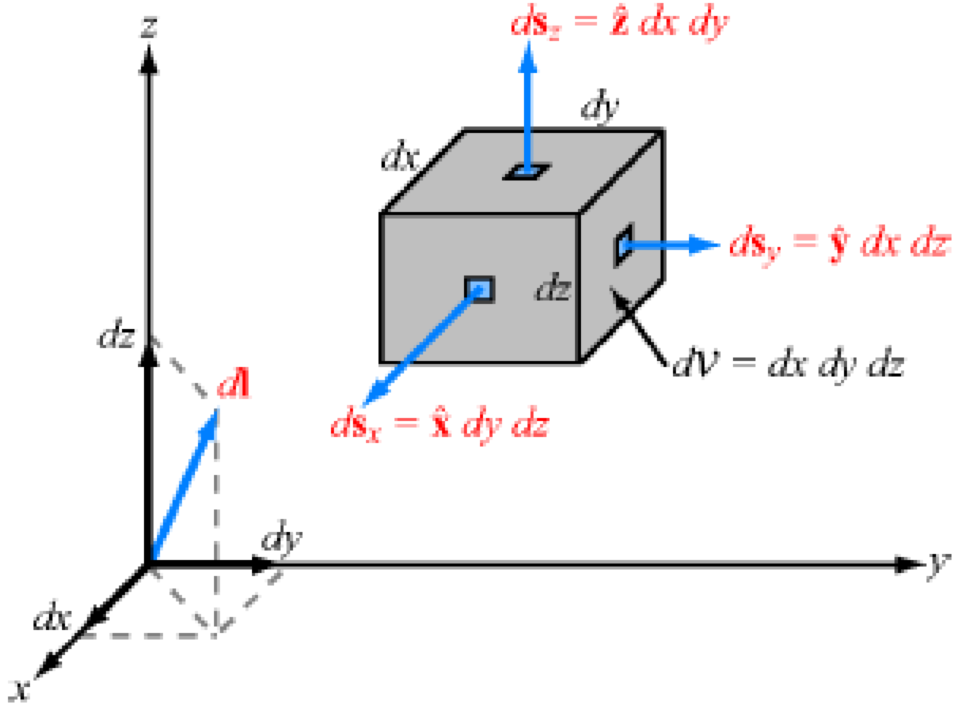
Coordinate Systems

Coordinate Systems

- Orthogonal
  - Cartesian
  - Cylindrical
  - Spherical
- Unorthogonal

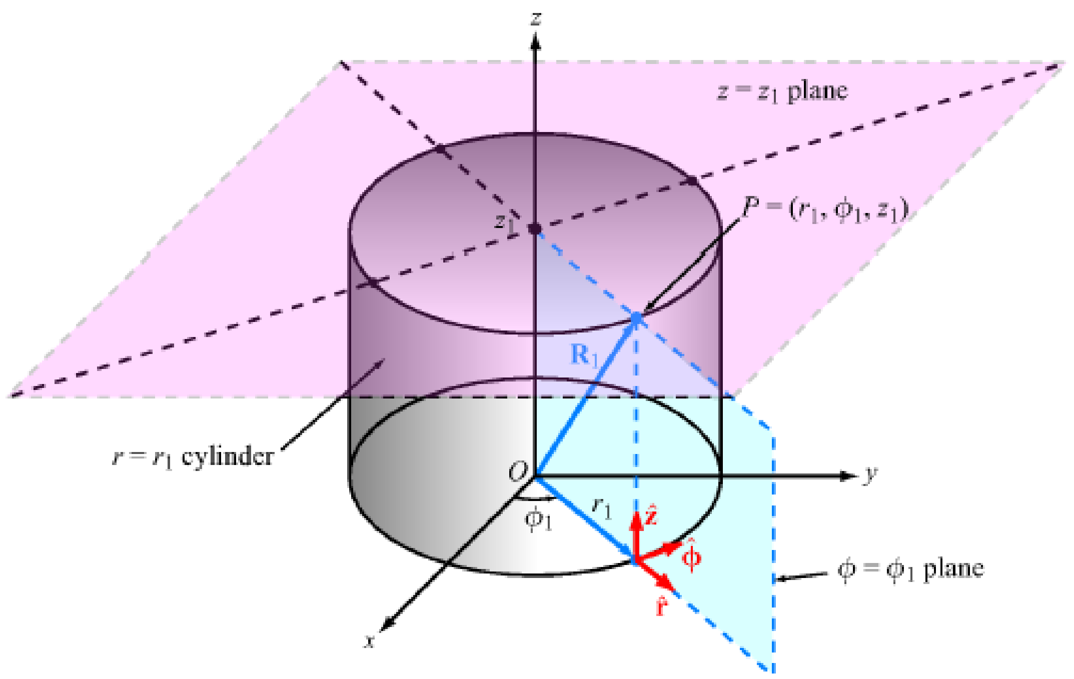
Cartesian Coordinate System

- Differential lengths
- Differential surface
- Differential volume



Cylindrical Coordinate System

- A point is defined by three variables $r, \emptyset, z$
- r is the radial distance from the z-axis
- $\emptyset$ is the azimuth angle as measured with reference to the positive x-axis
- z is the same as that in Cartesian coordinate system



Cylindrical Coordinate System

$$\hat{\mathbf{r}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}, \qquad \hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}, \qquad \hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$$

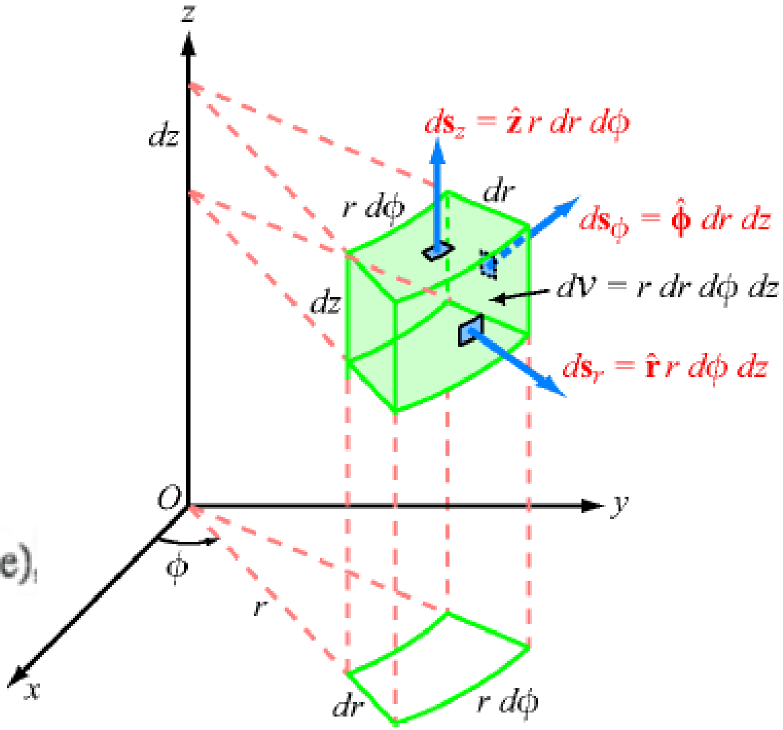
$$\hat{\mathbf{r}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \times \hat{\mathbf{z}} = 0.$$

$$\mathbf{A} = \hat{\mathbf{a}}|\mathbf{A}| = \hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$$

$$|\mathbf{A}| = \sqrt[+]{\mathbf{A} \cdot \mathbf{A}} = \sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}\,.$$

Cylindrical Coordinate System

$$dl_r = dr, \qquad dl_\phi = r\,d\phi, \qquad dl_z = dz.$$



$$d\mathbf{l} = \hat{\mathbf{r}}\,dl_r + \hat{\boldsymbol{\phi}}\,dl_\phi + \hat{\mathbf{z}}\,dl_z = \hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz.$$

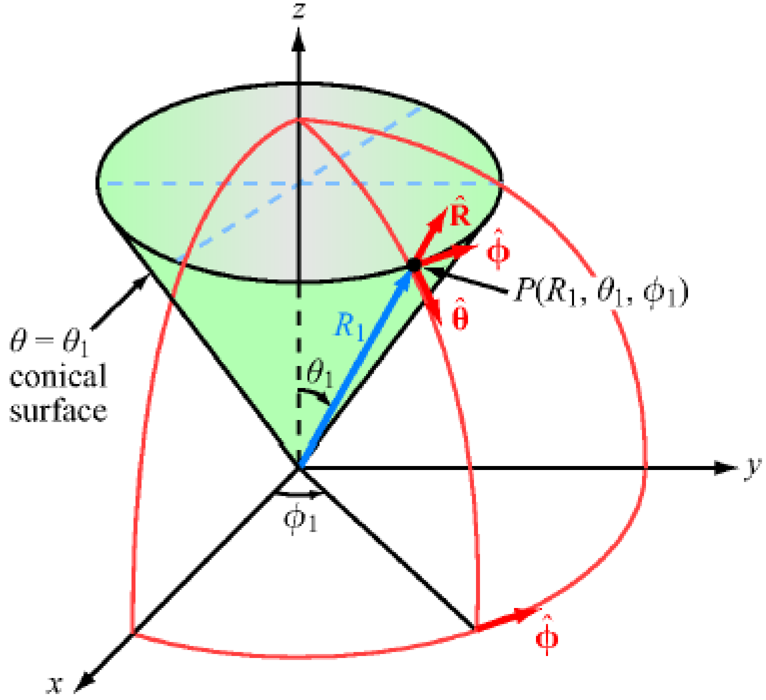
$$d\mathbf{s}_r = \hat{\mathbf{r}}\,dl_\phi\,dl_z = \hat{\mathbf{r}}r\,d\phi\,dz \quad (\phi\text{–}z \text{ cylindrical surface}),$$

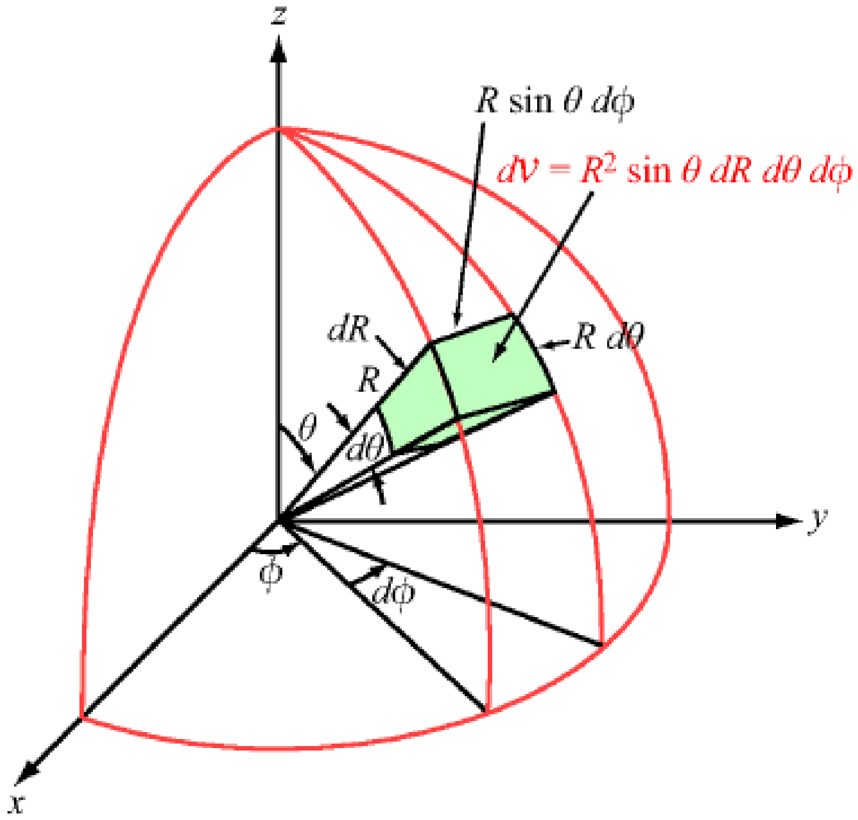
$$d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dl_r\,dl_z = \hat{\boldsymbol{\phi}}\,dr\,dz \quad (r\text{–}z \text{ plane}),$$

$$d\mathbf{s}_z = \hat{\mathbf{z}}\,dl_r\,dl_\phi = \hat{\mathbf{z}}r\,dr\,d\phi \quad (r\text{–}\phi \text{ plane}).$$

$$dV = dl_r\,dl_\phi\,dl_z = r\,dr\,d\phi\,dz.$$

Spherical Coordinate System





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